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Two Exact GMM Estimators**

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Estimating Euler Equations with Noisy Data: Two Exact GMM Estimators.¹

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Abstract

In this paper we exploit the specific structure of the Euler equation and develop two alternative GMM estimators that deal explicitly with measurement error. The first estimator assumes that the measurement error is lognormally distributed. The second estimator drops the distributional assumption and solves out for the unknown, but constant, conditional mean. Our monte carlo results suggest that both proposed estimators perform much better than conventional alternatives based on the exact Euler equation or its log-linear approximation, especially with short panels.

Keywords: Nonlinear models; Measurement error; Euler equation

JEL Classification: C13, E21

1 Introduction

Since Hall (1978) seminal paper, the Euler equation for consumption derived from the intertemporal problem faced by a generic consumer has played an important role in applied consumption research ¹. The reason for this is because it allows one to estimate structural preference parameters and possibly test the specification of the model without having to specify fully the stochastic environment in which the consumer operates. Unfortunately, the relationships that come out of most plausible preferences are highly non-linear. This creates a number of econometric problems, ranging from the performance of non-linear GMM estimators in small samples to the effect of measurement error (see Alan and Browning (2003) and Attanasio and Low (2004)) . The latter problem cannot be credibly ignored when working with microeconomic data, (see, for example, Shapiro (1984), Altonji and Siow (1987) and Runkle (1991)).

The standard non-linear GMM estimation yields inconsistent estimates in the presence of (neglected) measurement error (see, for instance, Amemiya (1985)). One solution that has been followed in the literature has been to work with a log linearized version of the Euler equation, a solution first used by Hansen and Singleton (1982). Such a procedure has several advantages, ranging from the robustness to the presence of classical measurement error, to the fact that it yields a specification linear in parameters and is therefore amenable to estimation using pseudo panel methods such as those proposed by Deaton (1985) and Browning, Deaton and Irish (1985). These advantages come however with two big problems. First and foremost, there are many situations in which log linearizing the Euler equation does not give a specification linear in parameters and therefore one is again subject to the same problems mentioned above. Situations of this type include a variety of interesting models, such as, for instance, models with

¹See Browning and Lusardi (1996) and Attanasio (1999).

habit formation. Second, the ‘constant’ of a log-linearized Euler equation includes conditional higher moments of consumption growth and interest rate, so that one loses the identifiability of the discount factor.

Moreover, if such conditional moments are not constant, the error term will include innovations to these moments that might be correlated with the instruments that one typically use in estimating the Euler equation. Although Attanasio and Low (2004) have shown that even in situations where the income process is heteroscedastic the log-linearized Euler equation seems to yield, if the time period is sufficiently large, decent estimates of the elasticity of intertemporal substitution, one would like to work with an estimator robust to these potential problems².

In the wider measurement error literature, resolutions of the problem for nonlinear estimators have only been possible in particular circumstances (see Hausman et al (1991), Hausman et al (1995), Schennach (2000), Hausman (2001), Wansbeek (2001), Hong and Tamer (2003)). In this paper we exploit the specific structure of the Euler equation to propose two alternative GMM estimators that deal explicitly with measurement error. Both assume that the measurement error is ‘classical’ in the sense of being stationary and multiplicative (but not necessarily with a unit mean). For the first estimator we additionally assume that the measurement error is lognormally distributed. This leads to a GMM in which the conditional expectation of the discounted change in the marginal utility is equal to a constant which is not equal to unity but which depends on the preference parameters and the variance of the measurement error³. The second estimator drops the distributional assumption and solves out for the unknown,

²Alan and Browning (2003) propose an alternative non Euler equation procedure which avoids the need to model any driving processes. This requires parameterising and simulating the martingale that expectations errors follow. Attanasio and Browning (1995) propose a different approach to the problem which consists in starting with a flexible but log-linear in parameters specification for the marginal utility of consumption. One then has to integrate such a specification to obtain the corresponding utility function.

³Colera (1994) follows the same parametric approach and assumes a lognormally distributed serially independent measurement error process. While her GMM estimator identifies the coefficient of relative risk aversion, discount factor and measurement error variance are not separately identified.

but constant, conditional mean⁴. The trade-off between the two estimators is the conventional one: the first estimator is more efficient if the errors are indeed lognormally distributed but the second estimator is more robust. We present the small sample behavior of our two GMM estimators as compared with the usual exact Euler equation GMM estimator and conventional log-linearized GMM.

Our Monte Carlo results suggest that both proposed estimators perform much better than conventional alternatives based on the exact Euler equation or its log-linear approximation, especially with short panels. As the sampling period increases, the bias in the coefficient of relative risk aversion parameter becomes smaller for both exact and log-linear Euler equation estimation whereas the bias in the discount factor does not respond to the panel length in the case of the exact Euler equation. The novel feature of our estimators is that they outperform the other two estimators in estimating both the coefficient of relative risk aversion and the discount factor especially when the available panel length is short. Both proposed estimators capture the true values of the coefficient of relative risk aversion and the discount factor remarkably well even in small samples and the resulting sampling distributions have reasonable dispersions.

2 The model and the proposed estimators.

In this section we discuss the estimation by GMM of a non-linear Euler equation for consumption where the consumption data are contaminated by measurement error. We will assume that we have enough time series observations and variation in intertemporal prices to be able to estimate the parameters of interest, which include the elasticity of intertemporal substitution and the discount factor. For expositional purposes, we consider the simplest version of a standard life

⁴Chioda (2003) follows a somewhat similar route, however, the GMM estimator she proposes fails to identify the discount factor.

cycle model with uncertainty. We could easily consider more complicated settings. A consumer maximizes expected utility. Utility is intertemporally additive and the instantaneous utility function is of the isoelastic form. Future utility is discounted at a rate $\beta < 1$. The individual consumer is not subject to liquidity constraints and has rational expectations. She can move resources over time by saving and borrowing in N assets with variable and stochastic returns. Her problem will then imply the following Euler equation for a generic asset i :

$$E_t \left[\left(\frac{c_{t+1}^*}{c_t^*} \right)^{-\gamma} (1 + r_{t+1}^i) \beta \right] = 1 \quad (1)$$

where c_t^* is consumption at time t , r_{t+1}^i the interest rate, β the discount factor and γ the coefficient of relative risk aversion. Such an equation can be used to estimate the preference parameters using non linear GMM. In order to implement GMM, we need a vector of instruments assumed to be included in the information set of the consumer. In this particular example, in the absence of measurement error a constant and a single instrument (such as the lagged real rate or lagged consumption growth) would deliver identification. For our discussion it is useful to express 1 as follows:

$$\left(\frac{c_{t+1}^*}{c_t^*} \right)^{-\gamma} (1 + r_{t+1}^i) \beta = \varepsilon_{t+1} \quad (2)$$

where ε_{t+1} is an expectational error uncorrelated with the information available at time t and, by definition, has unit conditional mean: $E_t(\varepsilon_{t+1}) = 1$.

Now suppose that *observed* consumption in time t , c_t^o , is observed subject to a multiplicative error: $c_t^o = c_t^* \eta_t$. Although we have chosen not to list the multiplicative assumption in our explicit list of assumptions, it is a critical assumption for identification. Consider the following

assumption for the measurement errors.

Assumption *The measurement error is stationary and independent of ‘everything’ (including lagged values of the measurement error and expectations errors, consumption levels and interest rates).*

Such an assumption is relatively weak and does *not* require that the measurement error have a unit mean. Neither does it require that measurement error has a specific distribution. We shall present two estimators that can be used to recover the structural parameters of this model under this assumption. If we write equation 2 in terms of observed consumption we will have

$$\left(\frac{c_{t+1}^o}{c_t^o}\right)^{-\gamma} (1 + r_{t+1})\beta = \varepsilon_{t+1} \left(\frac{\eta_{t+1}}{\eta_t}\right)^{-\gamma} \quad (3)$$

(where we now drop the i superscript for the asset return). Assumption 1 gives the following result for conditional expectations:

$$E_t \left(\varepsilon_{t+1} \left(\frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = E_t (\varepsilon_{t+1}) E_t \left(\left(\frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = E_t \left((\eta_{t+1})^{-\gamma} \right) E_t ((\eta_t)^\gamma) = \kappa \quad (4)$$

where the expectation is taken conditional to the information available at time t to the econometrician. This is obviously a subset of the information available to the agent, which presumably includes measurement error. However, the assumption that measurement error is independent of the expectational error can be used to justify the first equality in equation 4. The stationarity of the measurement equation gives that the conditional expectations of the measurement error do not depend on t . If the measurement error is constant ($\eta_t = \eta, \forall t$) then $\kappa = 1$. The exact value of κ depends on the distribution of the measurement error. If, for instance, we assume

that measurement error is log-normal, that is $\ln \eta_t \sim N(\mu, \nu)$, then we have:

$$\kappa = E_t \left(\left(\frac{\eta_{t+1}}{\eta_t} \right)^{-\gamma} \right) = \exp\{\gamma^2 \nu\} \quad (5)$$

Equation (4) implies that applying GMM to equation (3) ignoring measurement error would yield inconsistent estimates of both preference parameters. Notice that the mean of $\ln(\eta)$ does not enter this expression so that for this and subsequent results we do *not* have to assume that the measurement error has a unit mean.

In addition to the Euler equation (1) we can also consider the Euler equation which links consumption at t and at $t + 2$. Following the same steps used to derive equation (3), we can obtain:

$$\left(\frac{c_{t+2}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})(1 + r_{t+2})\beta^2 = \tilde{\varepsilon}_{t+2} \left(\frac{\eta_{t+2}}{\eta_t} \right)^{-\gamma} \quad (6)$$

The assumption of a stationary measurement error implies that $E_t \left(\left(\frac{\eta_{t+2}}{\eta_t} \right)^{-\gamma} \right) = \kappa$. Under the assumption of lognormality this term is also equal to $\exp\{\gamma^2 \nu\}$. Now, let's define u_{t+1}^1 and u_{t+2}^2 as follows:

$$\begin{aligned} u_{t+1}^1 &= \left[\left(\frac{c_{t+1}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})\beta - \kappa \right] \\ u_{t+2}^2 &= \left[\left(\frac{c_{t+2}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})(1 + r_{t+2})\beta^2 - \kappa \right] \end{aligned} \quad (7)$$

Notice that both 'residuals' in (7) have zero mean and are uncorrelated with information available at time t . A constant and a single instrument (for instance the lagged value of the interest

rate) would therefore deliver four orthogonality conditions that would over-identify the three parameters of the model $\{\beta, \gamma, \kappa\}$. If we denote with u_{t+2} the 2x1 vector containing u_{t+1}^1 and u_{t+2}^2 and with z_t the 2x1 vector containing a constant and a single instrument, the orthogonality conditions that GMM will use will be given by:

$$E \{u_{t+2} \otimes z_t\} = 0 \quad (8)$$

We refer to the GMM estimator that uses equation (8) as the GMM-LN estimator. If we are willing to assume log-normality, one can use estimates of κ to estimate the variance of measurement error using equation 5.

While (over) identification is delivered by a single instrument, one can potentially improve efficiency of this estimator by considering additional instruments. An important word of caution is necessary, however. An instrument that is typically used in the estimation of Consumption Euler equations is the lagged rate of growth of consumption. In the presence of measurement error such an instrument would be inappropriate. As it has become standard practice, one should lag such an instrument twice, to guarantee that is not correlated with u_{t+2} . Having said that, it should also be said that once lagged interest rates are valid instruments: there is no reason to believe that they are correlated with measurement error in consumption. Lagging them twice would cause an unnecessary loss of efficiency.

Our second estimator, uses the two equation in (7) to difference out κ and obtain orthogonality conditions that involve only β and γ . In particular, define:

$$v_{t+2}^1 = \left[\left(\frac{c_{t+1}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})\beta \right] - \left[\left(\frac{c_{t+2}^o}{c_t^o} \right)^{-\gamma} (1 + r_{t+1})(1 + r_{t+2})\beta^2 \right] \quad (9)$$

>From (7) is clear that v_{t+2}^1 has zero mean and is uncorrelated with observation available at $t - 1$ (subject to the same caveat that some variables like consumption growth should be lagged twice). This residual term can then be used to form orthogonality conditions to (over)identify the parameters of the model.

Notice that equation (9) does not depend on the parameters of the measurement error process. We refer to this estimator as GMM-D. As equation (9) is a form of double-differencing, we expect a significant decline in precision, as compared to the GMM-LN estimator.

In this section we have developed two GMM estimators for the consumption Euler equation if consumption is contaminated by multiplicative, stationary measurement error. The first estimator, GMM-LN (see equations (7) and (8)), makes use of the lognormality assumption. The second, (9), does not require any supplementary assumptions; we refer to it as the GMM-D estimator. We turn now to the small sample properties of four estimators: the linearized (approximate) GMM equation (AGMM), the exact GMM (EGMM, with no allowance for measurement error), GMM-LN and GMM-D.

3 Small Sample Results

The simulated data used in our experiments are generated by solving a standard intertemporal utility maximization problem under interest rate and labor income uncertainty. The details of the model and its solution are given in the Appendix. The parameter values for the coefficient of relative risk aversion and the discount factor are set to 4 and 0.95 respectively. The interest rate series for each agent is a stationary $AR(1)$ process with a coefficient of 0.6 and a mean of 0.05; thus the discount factor multiplied by the mean gross interest rate is very close to unity. We solve the model for 100 periods and drop the initial 19 and final 18 observations to avoid

starting and ending effects. For periods 20 to 81 the optimal consumption functions very close to identical and consumption growth is effectively stationary. To construct the data used in the experiments below we first generate consumption paths using the optimal program and then add multiplicative lognormal measurement error. The measurement error has a unit mean and a variance of 0.004, such that approximately 75% of the period to period variance in consumption growth is due to noise. This is at the upper end of estimates of the amount of noise in the real world data.

The experiments are performed to obtain the sampling distribution of the estimators developed in the previous section. For comparison, we also document the sampling distributions of conventional estimators based on the exact nonlinear Euler equation (EGMM) and its log-linear approximation (AGMM). We experiment with different panel lengths and we set the number of households for each estimation to 100. To be as close to real data availability as possible, we experiment with panel lengths of 15 and 25. We use instruments that are common in the empirical literature. Two sets of experiments are performed: The first set uses simulated data that are generated allowing for correlation of expectational errors across households. For this, we simply assume that every group of 100 households face the same interest rates and estimations are performed by pooling these 100 households together. The second set of experiments uses simulated data generated allowing for cross-sectional variation in interest rates so that every household faces different series (although the underlying interest rate process is the same for every household, ex-post series are different). Again, estimations are performed by pooling 100 households together. Notice that in both experiments individual households experience independent (across households) wage shocks. We set the number of Monte Carlo replications for both sets to 1000.

Table 1 presents the results for the first set of experiments in which the pooled households for a given estimation face the same interest rate series. The table presents the means, medians and standard deviations of the sampling distributions for all four estimators. Two persistent results immediately show in the table, the first obvious and the second somewhat surprising. First, all estimators improve in estimating the coefficient of relative risk aversion (γ) as sampling period (T) increases. Second, addition of twice lagged consumption growth (possibly a weak instrument) to the instrument set leads to severe downward bias in the estimates of γ for all exact (non-linear) estimators. The GMM on the log-linear Euler equation yields a mean value of γ that is quite close to the true value of 4 when T is 25 and the dispersion of the sampling distribution shrinks rapidly with T . Surprisingly, the exact GMM estimates γ reasonably well as T increases although as expected, discount factor (β) estimates display a strong downward bias and the bias does not seem to get smaller as T increases.

Both of our estimators GMM-LN and GMM-D do very well in estimating the intertemporal allocation parameters γ and β . For both estimators, as in the case of the conventional estimators, medians of sampling distributions are lower than means, less so for the first estimator (GMM-LN). With a panel length of 25, the first estimator (where we parameterize the measurement error distribution), yields a mean estimate for γ of 4. Even when the panel length is 15 the estimator performs much better than the conventional log-linear GMM and the exact GMM. The estimator captures the true value of β in both mean and median for both sampling periods. The mean of the second estimator (GMM-D) does equally well in capturing the true value of both γ and β for both panel lengths, however, as expected, the distribution of γ is now more dispersed compared to the first estimator.

Consequences of weak instruments in the case of a nonlinear GMM is a subject to an ongoing

research. Although it is well known that the weak identification leads conventional linear GMM to yield unreliable test statistics without affecting its consistency property, recent research points to a significant finite sample bias in such models. (see Hausman (2001), Stock and Wright (2000) and Stock, Wright and Yogo (2002)). Unfortunately, the finite sample consequences of weak identification in the case of a nonlinear GMM are virtually unknown. We do know that if an instrument is weak, correlation between the model error term and the instrument is close to zero even for the false values of the parameters of interest, leading to weak identification. This alone is enough to cause substantial finite sample bias in parameter estimates. Our Monte Carlo results strongly suggest a significant downward bias in the estimates of the coefficient of relative risk aversion for all three nonlinear estimators and the bias becomes smaller as T increases. However, unreported experiments suggest that even when T is unrealistically high (say, 1000) the downward bias in this parameter persists. Although addition of the twice lagged consumption growth to the instrument set does not change the performance of GMM-LN and GMM-D in estimating the discount factor, EGMM slightly improves with this additional instrument.

Table 2 presents the results for the second set of experiments. Recall that here pooled households face different interest rate series. Both of our estimators, GMM-LN and GMM-D, still perform better than the other two estimators and capture the true value of γ and β fairly well. The difference between our estimators and AGMM and EGMM is again substantial for small T . Both EGMM and AGMM display upward biases (in means) although both estimators greatly improve as T becomes large. Moreover, medians of the sampling distributions of γ for EGMM and AGMM are very close to the true value and this is not true for our proposed estimators. As was the case for the common interest rate experiments, both our new estimators

do an excellent job in capturing the true value of β , while EGMM seems hopeless. Finally, again, the addition of twice lagged consumption growth to the instrument set leads to a severe downward bias in the parameter γ for all non-linear estimators while leaving β unaffected (it only slightly improves the β estimates of EGMM).

Overall, the results suggest that the type of measurement error assumed in the literature causes a serious downward bias in the discount factor estimates if it is ignored and this bias does not go away as the panel length increases. The bias in the coefficient of relative risk aversion is not as severe and it seems to become smaller as the panel length increases (particularly in the common interest rate case). It is quite interesting that with a short panel of 15 years, the log-linearized Euler equation yields almost the same result as the exact one, whether interest rate series are common or there is a cross section variation. Recall that the main reason for using the log-linearized version of the Euler equation is that linearizing the exact Euler equation removes the measurement error by shifting it to the expectation errors. The log-linear version seems to be superior to the exact version only as the panel length becomes (unrealistically) long.

Turning to our estimators, they both perform very well in estimating the parameters of interest. The most striking feature of our estimators is that they outperform the other two estimators especially when the panel length is short. This is clearly an important feature since panel data on consumption are rare, let alone long panels. These estimators' advantage over traditional approaches is more pronounced in the case where households face the same interest rate series (which is perhaps a more realistic assumption than cross sectional variation in interest rates).

4 Conclusion

In this paper we have proposed two new GMM estimators for the Euler equation that take account of the measurement error in consumption. A feature of dealing with measurement error in nonlinear models is that the solutions are specific to the context and that is the case here. In particular, we have exploited the iso-elastic assumption which gives a ratio of consumptions in the Euler equation. If some other functional form used then different assumptions will be needed to achieve identification. Monte Carlo simulations suggest that when the panel length is short both of our estimators perform significantly better than the linearized Euler equation or the exact GMM estimator that ignores measurement error. The estimator that makes a specific lognormal assumption, GMM-LN, performs better when that assumption is correct but is likely to be less robust than the estimator that simply assumes stationarity, GMM-D.

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		γ		β		σ^2	
Estimator	Instrument	$T = 15$	$T = 25$	$T = 15$	$T = 25$	$T = 15$	$T = 25$
AGMM	$\ln(1 + r_t)$	2.43	3.76	—	—	—	—
		[2.54]	[3.29]	—	—	—	—
		(25.1)	(3.10)	—	—	—	—
AGMM	$\ln(1 + r_t), \ln(\frac{C_{t-1}}{C_{t-2}})$	3.67	3.77	—	—	—	—
		[2.56]	[3.28]	—	—	—	—
		(43.8)	(3.12)	—	—	—	—
EGMM	$(1 + r_t)$	2.92	3.59	0.90	0.89	—	—
		[2.52]	[3.22]	[0.92]	[0.91]	—	—
		(2.89)	(2.31)	(0.09)	(0.08)	—	—
EGMM	$(1 + r_t), (\frac{C_{t-1}}{C_{t-2}})$	2.02	2.83	0.92	0.92	—	—
		[1.76]	[2.47]	[0.94]	[0.93]	—	—
		(1.87)	(1.78)	(0.05)	(0.05)	—	—
GMM-LN	$(1 + r_t)$	3.68	4.00	0.95	0.95	0.006	0.003
		[3.56]	[3.88]	[0.95]	[0.95]	[0.004]	[0.004]
		(2.43)	(2.29)	(0.01)	(0.01)	(0.03)	(0.03)
GMM-LN	$(1 + r_t), (\frac{C_{t-1}}{C_{t-2}})$	1.98	3.07	0.95	0.95	0.004	0.004
		[1.83]	[2.92]	[0.95]	[0.95]	[0.004]	[0.004]
		(1.72)	(1.76)	(0.01)	(0.008)	(0.01)	(0.01)
GMM-D	$(1 + r_t)$	4.05	4.04	0.95	0.95	—	—
		[2.82]	[3.43]	[0.95]	[0.95]	—	—
		(3.93)	(2.77)	(0.02)	(0.01)	—	—
GMM-D	$(1 + r_t), (\frac{C_{t-1}}{C_{t-2}})$	2.16	2.82	0.95	0.95	—	—
		[1.75]	[2.51]	[0.95]	[0.95]	—	—
		(2.10)	(1.83)	(0.01)	(0.01)	—	—
Note: Values are means, medians (in square brackets) and standard deviations (in brackets) of sampling distributions. $N = 100$. True values: $\gamma = 4$, $\beta = 0.95$, $\sigma^2 = 0.004$							

Table 1: Monte Carlo Results: Common Interest Rates

		γ		β		σ^2	
Estimator	Instrument	$T = 15$	$T = 25$	$T = 15$	$T = 25$	$T = 15$	$T = 25$
AGMM	$\ln(1 + r_t)$	4.65	4.38	—	—	—	—
		[4.01]	[3.97]	—	—	—	—
		(9.72)	(1.66)	—	—	—	—
AGMM	$\ln(1 + r_t), \ln(\frac{C_{t-1}}{C_{t-2}})$	7.65	4.38	—	—	—	—
		[4.01]	[3.98]	—	—	—	—
		(13.6)	(1.67)	—	—	—	—
EGMM	$(1 + r_t)$	4.60	4.38	0.87	0.88	—	—
		[4.07]	[3.94]	[0.89]	[0.89]	—	—
		(2.04)	(1.68)	(0.08)	(0.07)	—	—
EGMM	$(1 + r_t), (\frac{C_{t-1}}{C_{t-2}})$	3.18	3.54	0.91	0.90	—	—
		[2.94]	[3.31]	[0.92]	[0.91]	—	—
		(1.50)	(1.28)	(0.05)	(0.04)	—	—
GMM-LN	$(1 + r_t)$	3.83	3.88	0.95	0.95	0.004	0.004
		[3.60]	[3.71]	[0.95]	0.95	[0.004]	0.004
		(1.18)	(0.98)	(0.004)	(0.01)	(0.001)	(0.001)
GMM-LN	$(1 + r_t), (\frac{C_{t-1}}{C_{t-2}})$	3.06	3.45	0.95	0.95	0.00	0.004
		[2.87]	[3.26]	[0.95]	[0.95]	[0.004]	[0.004]
		(1.38)	(1.19)	(0.003)	(0.003)	(0.14)	(0.001)
GMM-D	$(1 + r_t)$	4.32	3.75	0.95	0.95	—	—
		[3.86]	[3.53]	[0.95]	[0.95]	—	—
		(1.83)	(1.05)	(0.003)	(0.002)	—	—
GMM-D	$(1 + r_t), (\frac{C_{t-1}}{C_{t-2}})$	3.07	3.39	0.95	0.95	—	—
		[2.84]	[3.21]	[0.95]	[0.95]	—	—
		(1.35)	(1.13)	(0.004)	(0.003)	—	—
Note: Values are means, medians (in square brackets) and standard deviations (in brackets) of sampling distributions. $N = 100$. True values: $\gamma = 4$, $\beta = 0.95$, $\sigma^2 = 0.004$							

Table 2: Monte Carlo Results: Cross Section Interest Rate Variation

A The Simulation Model.

We assume that the utility function is intertemporally additive and the sub-utilities are iso-elastic. The problem of the generic household is:

$$\max_{c_t} E_t \sum_{t=0}^T \beta^t u(C_t)$$

$$s.t. X_{t+1} = (1 + r_{t+1})(X_t - C_t) + Y_{t+1}$$

where C_t is nondurable consumption in period t , X_t is cash-on-hand (total financial and non-financial wealth) and Y_t is current labor income. We assume that durable consumption and leisure are separable from the nondurable consumption. The income process is assumed as follows:

$$Y_{t+1} = P_{t+1}U_{t+1}$$

$$P_{t+1} = GP_tN_{t+1}$$

P_t is permanent income which is subject to lognormally distributed shocks N_t with mean unity and variance $\exp(\sigma_n^2) - 1$, current income Y_t equals permanent income multiplied by a transitory shock which is distributed lognormally with mean one and variance $\exp(\sigma_u^2) - 1$. We assume income growth G to be nonstochastic and equal to 1.

Interest rate series are assumed to be generated by a stationary first order autoregressive process with long-run mean μ and autoregressive coefficient ρ . Interest rates shocks ϵ_{t+1} are

assumed to be white noise with variance σ_ϵ^2 . The process is

$$r_{t+1} = (1 - \rho)\mu + \rho r_t + \epsilon_{t+1}$$

The intertemporal model described above does not have an analytical solution due to the assumed income uncertainty. Therefore we utilized the standard numerical dynamic programming methods to obtain a solution. Since the utility function is additive over the life cycle we solved the model recursively starting from the last period of life. We assume away any bequest motive so that consumption in period T is:

$$c_T(x_T) = x_T$$

The problem is solved via policy function iteration using the terminal value condition. Having a nonstationary income process makes the problem harder to solve since the range of possible income values is too large. Instead, we redefine all relevant variables in terms of their ratios to permanent income and solve for the consumption to income ratio. By doing this we reduced the number of state variables to two, namely the cash on hand to income ratio and the interest rate. Moreover, we obtain an iid income process which can be approximated by standard Quadrature methods. Given the redefinition of the variables, the Euler equation can be written as

$$\theta_t(w_t, r_t)^{-\gamma} - \frac{1}{(1 + \delta)} E_t \left[(1 + r_{t+1}) \theta_{t+1}(w_{t+1}, r_{t+1})^{-\gamma} n_{t+1}^{-\gamma} \right] = 0$$

where $\theta_t = \frac{C_t}{P_t}$, $w_t = \frac{X_t}{P_t}$. At the terminal date T , consumption to income ratio is a function of only the cash on hand to income ratio and since the bequest motive is assumed away it follows that $\theta_T = w_T$.

For the income process, we use 10 point Gaussian Quadrature and we approximate the interest rate process by forming a 10 point first order discrete Markov process. We use a cubic spline to approximate the consumption function at each iteration. The agent is allowed to borrow the amount he can pay back with certainty. In practice this constraint will never bind because the functional form of the utility function implies that zero consumption results in infinite marginal utility. Since we do not assume an explicit borrowing limit, the consumption functions are continuously differentiable. In fact, in our case where agents have iso-elastic preferences and income uncertainty, consumption functions are strictly concave.

In order to solve the problem, we define an exogenous grid for the cash on hand to income ratio: $\{x_j\}_{j=1}^J$. It is important to adjust the grid as the solution goes back in time. The algorithm finds the consumption level that makes the standard Euler equation hold for each value of x and r . In practice, we took 100 points for x and 10 points for r . The grid for x is finer at lower levels in order to capture the curvature of the consumption function. After solving for the consumption function of a generic household for 100 periods, we simulate consumption paths for 100,000 ex-ante identical households facing different interest rate realizations. Then, using the same consumption function, we simulate paths for 100,000 ex-ante identical households where groups of 100 households face the same realized interest rate series. We use only 15 and 25 periods for the estimations and these observations are taken from the middle of each path. Table 3 presents the assumed parameter values for our experiments. The discount rate and the mean interest rate are chosen to be close in order to prevent consumers to quickly go towards the borrowing constraint⁵.

⁵When the discount rate is large relative to the interest rate, consumers borrow close to the maximum possible amount. Then the movement of consumption is largely driven by income and the identification of interest rate impact on consumption growth becomes very difficult. See Attanasio and Low (2004) and Alan and Browning (2003) for a detailed discussion on the problems of identification and impatience.

Parameter	Value
Coefficient of risk aversion (γ)	4
Discount factor (β)	.95
Discount rate ($\delta = \frac{1}{\beta} - 1$)	.053
Standard deviation of permanent income shocks (σ_n)	.02
Standard deviation of transitory income shocks (σ_u)	.1
Unconditional mean of interest rate process (μ)	.05
AR(1) coefficient of interest rate process (ρ)	.6
Standard deviation of interest rate process (σ_ϵ)	.025
Variance of log measurement error (σ_η)	.004 (75% noise)

Table 3: Parameter Values